NPRE 446: Interaction of Radiation with Matter
Homework Assignments

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1 Homework 1

Attention:

1. Write down your UIN instead of your name if you worry about privacy when turning in homework.

2. Explanation of the score: our brains typically consume about 0.2 Calories per minute. When actively thinking, our brains can kick it up to burning about 1 Calorie per minute. So instead of assigning each question with points, I will assign with Calories. For example, if a problem is given 10 Calories, it means you will need to burn about 10 Calories to solve the problem and the estimated time to solve the problem is about 10 minutes.

3. Because of the breath and depth of the content of the course, it is only possible to cover the essence during the lectures. One must read the relevant chapters in the textbooks to learn the details and gain deeper understandings.

Readings:


1.1

What are the energies of an electron, a proton, a neutron, and a photon with wavelengths $\lambda = 1 \, \text{Å}$ and 1 fm, respectively? (20 Calories)

1.2

In Bohr’s model of hydrogen, the electron in its ground state was supposed to travel in a circle of radius $5 \times 10^{-11} \, \text{m} = 0.5 \, \text{Å}$, held in orbit by the Coulomb attraction of the proton. According to classical electrodynamics, this electron should radiate, and hence spiral into the nucleus. Show that $v \ll c$ for most of the trip (so you can use the Larmor formula), and calculate the lifespan of Bohr’s atom. (Assume each revolution is essentially circular.) [Griffiths’s Electrodynamics: Page 487, Question 11.14] (30 Calories)

Notes: The Larmor formula of the radiation power of a moving charged particle with an acceleration $a$ is:

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$m_e = 9.11 \times 10^{-31} \, \text{kg}$
$q_e = 1.6 \times 10^{-19} \, \text{C}$
$\mu_0 = 4\pi \times 10^{-7} \, \text{N/A}^2$
$g = 9.8 \, \text{m/s}^2$
$c = 3.0 \times 10^8 \, \text{m/s}$

1.3

Griffiths’s Quantum Mechanics: Page 20, Problem 1.9. (30 Calories)
1.4

Griffiths's Quantum Mechanics: Page 22, Problem 1.17. (30 Calories)
2 Homework 2

Readings:

2.1
Griffiths’s Quantum Mechanics: Page 23, Problem 1.18. (30 Calories)

2.2
Consider a particle in a one dimensional infinite square well potential:

\[ V(x) = \begin{cases} 
0, & 0 \leq x \leq a \\
\infty, & \text{otherwise} 
\end{cases} \]

1. Compute the eigen energies and the eigen state wave functions. (20 Calories)

2. Compute \( \langle x \rangle \), \( \langle x^2 \rangle \), \( \sigma_x \), \( \langle p \rangle \), \( \langle p^2 \rangle \), \( \sigma_p \) for the \( n \)-th eigen state. Compute the uncertainty relation quantity \( \sigma_x \sigma_p \). Check whether the uncertainty relation is satisfied. Which state is closest to the uncertainty limit? (20 Calories)

3. If the initial wave function is

\[ \Psi(x,0) = \begin{cases} 
Ax, & 0 \leq x \leq \frac{a}{2} \\
A(a - x), & \frac{a}{2} \leq x \leq a \\
0, & \text{otherwise} 
\end{cases} \]

for some constant \( A \).

(a) Sketch \( \Psi(x,0) \). Determine the constant \( A \). (10 Calories)

(b) Compute \( \Psi(x,t) \). (20 Calories)

(c) If we perform a measurement of the energy, what values may we get and with what probabilities? What’s the expectation value? (10 Calories)

2.3
Griffiths’s Quantum Mechanics: Page 38, Problem 2.5. (30 Calories)
3 Homework 3

Readings:

3.1
Griffiths's Quantum Mechanics: Page 85, Problem 2.36. (30 Calories)

3.2
Griffiths's Quantum Mechanics: Page 67, Problem 2.21. (30 Calories)

3.3
Griffiths’s Quantum Mechanics: Page 83, Problem 2.33. (30 Calories)
4 Homework 4

Readings:

4.1
Griffiths’s Quantum Mechanics: Page 83, Problem 2.34. (40 Calories)

4.2
Griffiths’s Quantum Mechanics: Page 84, Problem 2.35. (30 Calories)
5 Homework 5

Readings:

5.1
What are the kinetic energy, velocity, and wavelength of thermal \( T = 300 \text{ K, room temperature} \) neutrons? (15 Calories) Why are thermal neutrons useful for materials studies? Give at least three reasons. (5 Calories)

5.2
What is the radius of gyration \( R_g \) of a uniformly distributed spherical nucleus with a radius \( R \)?
Note: \( R_g = \langle r^2 \rangle^{\frac{1}{2}} = (\int \rho(r) r^2 dV)^{\frac{1}{2}}. \) (10 Calories)

5.3
What is the potential \( V(r) \) of an electron inside and outside a uniformly distributed spherical nucleus with a radius \( R \) and charge \( Ze \)? (20 Calories)

5.4
1. Sketch the mass or charge distribution of a typical nucleus and explain the main features of the curve. (10 Calories)
2. What are isotopes, isobars, and isotones? Give one example of each of them. (10 Calories)
3. What is binding energy \( B(A,Z) \)? (5 Calories)
4. Sketch the binding energy per nucleon \( B/A \) vs \( A \) and explain the main features of the curve. (10 Calories)

5.5
1. Compute the binding energy per nucleon \( B/A \) for \(^4\text{He}, ^7\text{Li}, ^{56}\text{Fe}, \) and \(^{238}\text{U} \). (10 Calories)
2. Compute the neutron separation energy \( S_n \) for \(^4\text{He}, ^7\text{Li}, ^{56}\text{Fe}, \) and \(^{238}\text{U} \). (10 Calories)
3. Compute the proton separation energy \( S_p \) for \(^4\text{He}, ^7\text{Li}, ^{56}\text{Fe}, \) and \(^{238}\text{U} \). (10 Calories)