1 Homework 1

Attention:

1. Write down your UIN instead of your name if you worry about privacy when turning in homework. Also write down the course number next to your UIN clearly.

2. Explanation of the score: our brains typically consume about 0.2 Calories per minute. When actively thinking, our brains can kick it up to burning about 1 Calorie per minute. So instead of assigning each question with points, I will assign with Calories. For example, if a problem is given 10 Calories, it means you will need to burn about 10 Calories to solve the problem and the estimated time to solve the problem is about 10 minutes.

3. Because of the breath and depth of the content of the course, it is only possible to cover the essence during the lectures. One must read the relevant chapters in the textbooks to learn the details and gain deeper understandings.

Readings:


1.1 Griffiths’s Quantum Mechanics: Page 14, Problem 1.5. (30 Calories)

1.2 Griffiths’s Quantum Mechanics: Page 21, Problem 1.14. (30 Calories)

1.3 Griffiths’s Quantum Mechanics: Page 22, Problem 1.15. (30 Calories)
2 Homework 2

Readings:

2.1

1. From the continuity equation
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot J = 0 \]
derive the probability current
\[ J = \frac{i}{2m} (\Psi \nabla \Psi^* - \Psi^* \nabla \Psi) = \frac{\hbar}{m} \text{Im} \{ \Psi^* \nabla \Psi \} \]
(10 Calories)

2. Compute the probability current for the plain wave \( \Psi(x,t) = e^{i(kx-\omega t)} \). (10 Calories)

3. Compute the probability current for the spherical wave \( \Psi(r,t) = e^{i(kr-\omega t)/r} \). (10 Calories)

2.2

The Hamiltonian of a 1-dimensional quantum harmonic oscillator is
\[ H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2 \]

1. Solve the stationary Schrödinger equation to show the eigen state wave functions \( \psi_n(x) \) and the corresponding eigen energy \( E_n \) are
\[ \psi_n(x) = \frac{1}{\sqrt{2^n n!}} \left( \frac{\lambda}{\pi} \right)^{1/4} \exp \left( -\frac{1}{2} \lambda x^2 \right) H_n \left( \sqrt{\lambda} x \right) \]
\[ E_n = \left( n + \frac{1}{2} \right) \hbar \omega \]
where \( \lambda = m \omega / \hbar \), \( H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2} \) are the Hermite polynomials. (30 Calories)

2. For \( n = 0, 1, 2 \), write down \( \psi_n(x) \) explicitly, produce computer plots of the wave functions \( \psi_n(x) \) and the square of the wave functions \( |\psi_n(x)|^2 \) on top of the potential. Attach your code. (15 Calories)

3. For \( n = 0, 1, 2 \), check the orthogonality of \( \psi_n(x) \) by explicit integrations. (15 Calories)

4. For \( n = 0, 1, 2 \), compute \( \langle x \rangle \), \( \langle x^2 \rangle \), \( \sigma_x \), \( \langle p \rangle \), \( \langle p^2 \rangle \), \( \sigma_p \). Compute the uncertainty relation quantity \( \sigma_x \sigma_p \). Check whether the uncertainty relation is satisfied. Which state is closest to the uncertainty limit? (15 Calories)
5. For $n = 0, 1, 2$, compute the expectation values of the kinetic energy $\langle T \rangle$ and the potential energy $\langle V \rangle$ by explicit integrations. (15 Calories)

6. If the particle starts out in the initial state

$$\Psi(x, 0) = A \left[ 3\psi_0(x) + 4\psi_1(x) \right]$$

what is $A$? what is $\Psi(x, t)$? If we measure the energy of this particle, what values may we get and with what probabilities? What’s the expectation value of $\langle H \rangle$? (20 Calories)