NPRE 447 & 521 Interaction of Radiation with Matter II
Homework Assignments

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1 Homework 1

Attention:

1. Both 447 and 521 students should solve the problems without asterisks. In addition, 521 students should also solve the problems marked with asterisks. 447 students will not get extra credit for solving the problems with asterisks.

2. Write down your UIN instead of your name if you worry about privacy when turning in homework. Also write down the course number next to your UIN clearly.

3. Explanation of the score: our brains typically consume about 0.2 Calories per minute. When actively thinking, our brains can kick it up to burning about 1 Calorie per minute. So instead of assigning each question with points, I will assign with Calories. For example, if a problem is given 10 Calories, it means you will need to burn about 10 Calories to solve the problem and the estimated time to solve the problem is about 10 minutes.

4. Because of the breath and depth of the content of the course, it is only possible to cover the essence during the lectures. One must read the relevant chapters in the textbooks to learn the details and gain deeper understandings.

Readings:


1.1

In Bohr’s theory of hydrogen, the electron in its ground state was supposed to travel in a circle of radius $5 \times 10^{-11} \text{ m}$, held in orbit by the Coulomb attraction of the proton. According to classical electrodynamics, this electron should radiate, and hence spiral into the nucleus. Show that $v \ll c$ for most of the trip (so you can use the Larmor formula), and calculate the lifespan of Bohr’s atom. (Assume each revolution is essentially circular.) [Griffiths’s Electrodynamics: Page 487, Question 11.14] (30 Calories)

1.2

Griffiths’s Quantum Mechanics: Page 20, Problem 1.9. (30 Calories)

1.3

Griffiths’s Quantum Mechanics: Page 22, Problem 1.17. (30 Calories)

1.4

Griffiths’s Quantum Mechanics: Page 23, Problem 1.18. (30 Calories)
1.5  *
Griffiths’s Quantum Mechanics: Page 22, Problem 1.15. (30 Calories)

1.6  *
The Hamiltonian of a 1-dimensional quantum harmonic oscillator is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

Its eigen wave functions \(\psi_n(x)\) and the corresponding eigen energy \(E_n\) are

$$\psi_n(x) = \frac{1}{\sqrt{2^{n!}n!}} \left( \frac{\lambda}{\pi} \right)^{1/4} \exp \left( -\frac{1}{2} \lambda x^2 \right) H_n(\sqrt{\lambda} x)$$

$$E_n = \left( n + \frac{1}{2} \right) \hbar \omega$$

where \(\lambda = m\omega/\hbar\), \(H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}\) are the Hermite polynomials.

1. For \(n = 0, 1, 2\), find out and sketch the wave functions \(\psi_n(x)\) and the square of the wave functions \(|\psi_n(x)|^2\) on top of the potential. (10 Calories)

2. For \(n = 0, 1, 2\), check the orthogonality of \(\psi_n(x)\) by explicit integration. (15 Calories)

3. For \(n = 0, 1, 2\), compute \(\langle x \rangle\), \(\langle x^2 \rangle\), \(\sigma_x\), \(\langle p \rangle\), \(\langle p^2 \rangle\), \(\sigma_p\). Compute the uncertainty relation quantity \(\sigma_x \sigma_p\). Check whether the uncertainty relation is satisfied. Which state is closest to the uncertainty limit? (15 Calories)

4. For \(n = 0, 1, 2\), compute the expectation values of the kinetic energy \(\langle T \rangle\) and the potential energy \(\langle V \rangle\). (15 Calories)

5. If the particle starts out in the initial state

$$\Psi(x, 0) = A \left[ 3\psi_0(x) + 4\psi_1(x) \right]$$

what is \(A\)? what is \(\Psi(x, t)\)? If we measure the energy of this particle, what values may we get and with what probabilities? What’s the expectation value of \(\langle H \rangle\)? (15 Calories)
# 2 Homework 2

**Readings:**


## 2.1

Consider a particle in a one dimensional infinite square well potential:

\[ V(x) = \begin{cases} 
0, & 0 \leq x \leq a \\
\infty, & \text{otherwise} 
\end{cases} \]

1. Compute the eigen energies and the eigen state wave functions. (20 Calories)

2. Compute \( \langle x \rangle \), \( \langle x^2 \rangle \), \( \sigma_x \), \( \langle p \rangle \), \( \langle p^2 \rangle \), \( \sigma_p \) for the \( n \)-th eigen state. Compute the uncertainty relation quantity \( \sigma_x \sigma_p \). Check whether the uncertainty relation is satisfied. Which state is closest to the uncertainty limit? (20 Calories)

3. If the initial wave function is

\[ \Psi(x, 0) = \begin{cases} 
Ax, & 0 \leq x \leq \frac{a}{2} \\
A(a - x), & \frac{a}{2} \leq x \leq a \\
0, & \text{otherwise} 
\end{cases} \]

for some constant \( A \).

(a) Sketch \( \Psi(x, 0) \). Determine the constant \( A \). (10 Calories)

(b) Compute \( \Psi(x, t) \). (20 Calories)

(c) If we perform a measurement of the energy, what values may we get and with what probabilities? What’s the expectation value? (10 Calories)

## 2.2

Griffiths’s Quantum Mechanics: Page 38, Problem 2.5. (30 Calories)

## 2.3

Griffiths’s Quantum Mechanics: Page 84, Problem 2.35. (30 Calories)

## 2.4 *

In atomic and nuclear physics, many systems can be described by pseudo-potentials, such as the Fermi pseudo-potential \( V(r) = \frac{2\pi \hbar^2}{m} b \delta(r) \) used in describing neutron scattering processes, where \( b \) is the bound scattering length. Let’s consider a particle with mass \( m \) in the 1-dimensional \( \delta \)-potential

\[ V(x) = -V_0 \delta(x) \]

where \( V_0 > 0 \) is the strength of the potential with the dimension of [EL].
1. Compute the bound state \((E < 0)\) wave function and the corresponding eigen energy level(s). (30 Calories)

2. Compute the probability current of the wave function in Part 1. Explain the physical meaning of the result. (5 Calories)

3. Compute the transmitted and reflected wave functions of an incoming plane wave \(\psi_i(x) = Ae^{ikx} \) \((E > 0\), scattering state\). (30 Calories)

4. Compute the probability current on both sides of the potential in Part 3. Explain the physical meaning of the result. (10 Calories)

5. Compute the transmission coefficient \(T\) and reflection coefficient \(R\) in Part 3. (10 Calories)

6. Perform dimensional analysis on the eigen energy computed in Part 1, and the transmission and reflection coefficients computed in Part 5. (10 Calories)

7. What’s the asymptotic behavior of \(T\) and \(R\) if the potential is very deep, i.e. \(V_0 \to \infty\)? Explain the physical meaning. (5 Calories)
3  Homework 3

Readings:

3.1
1. From the continuity equation
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot J = 0 \]
derive the probability current
\[ J = \frac{i\hbar}{2m}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi) = \frac{\hbar}{m}\text{Im}\{\Psi^*\nabla\Psi\} \]  
(10 Calories)
2. Compute the probability current for the plain wave \( \Psi(x,t) = e^{i(kx-\omega t)} \). (10 Calories)
3. Compute the probability current for the spherical wave \( \Psi(r,t) = e^{i(kr-\omega t)}/r \). (10 Calories)

3.2
Griffiths’s Quantum Mechanics: Page 85, Problem 2.36. (30 Calories)

3.3
Griffiths’s Quantum Mechanics: Page 83, Problem 2.33. (30 Calories)

3.4
Macroscopic Quantum World The Planck constant (reduced) \( \hbar \approx 1 \times 10^{-34} \text{J} \cdot \text{s} \) plays a fundamental role in quantum mechanics. Imagine that one day you are teletransported to another universe, where the reduced Planck constant is \( 10^{34} \) times larger, i.e. \( \hbar \approx 1 \text{J} \cdot \text{s} \). Let’s picture what strange phenomena you would expect.

1. (*) First, derive the uncertainty principle
\[ \sigma_A\sigma_B \geq \frac{1}{2} |\langle [A,B]\rangle| \]
and state its the significance. (10 Calories)
2. Suppose you have a jar of candies in this quantum world. When you open the jar, estimate the escape velocities of the candies. Do you need to be careful when opening the jar? (A typical weight of a candy is 1 gram. A typical size of a jar is 10 cm.) (10 Calories)
3. Use your imagination, describe another two strange phenomena you would expect in this quantum world. Be as quantitative as possible. (20 Calories)
3.5 *

Let’s reconsider the 1-dimensional quantum harmonic oscillator

\[ H = T + V = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 \]

using the ladder operators

\[ a = \sqrt{\frac{\hbar m\omega}{2}} (x + \frac{ip}{m\omega}) \]
\[ a^\dagger = \sqrt{\frac{\hbar m\omega}{2}} (x - \frac{ip}{m\omega}) \]

1. Solve \( x \) and \( p \) in terms of \( a \) and \( a^\dagger \). Show that the Hamiltonian operator can be written in terms of the ladder operators \( H = (N + \frac{1}{2})\hbar\omega \), where \( N = a^\dagger a \). (10 Calories)

2. Compute \([a, a^\dagger]\), \([N, a]\), and \([N, a^\dagger]\), where \( N = a^\dagger a \). (15 Calories)

3. Show that the energy eigenstates are \(|n\rangle = \frac{1}{\sqrt{n!}} (a^\dagger)^n |0\rangle\) and the energy eigenvalues are \( E_n = (n + \frac{1}{2})\hbar\omega \). (20 Calories)

4. Sketch the wave functions of the first three eigen states \((n = 0, 1, \text{ and } 2)\) in a harmonic potential. (15 Calories)

5. For the \( n^{\text{th}} \) energy eigenstate \(|n\rangle\), compute the expectation values of \( \langle n|x|n\rangle \), \( \langle n|x^2|n\rangle \), \( \langle n|p|n\rangle \), \( \langle n|p^2|n\rangle \), and the uncertainty relation quantity \( \sigma_x \sigma_p \). (10 Calories)

6. For the \( n^{\text{th}} \) energy eigenstate \(|n\rangle\), compute the expectation values of the kinetic energy \( \langle n|T|n\rangle \) and the potential energy \( \langle n|V|n\rangle \). (10 Calories)

7. If the particle starts out in the initial state

\[ |\Psi(t = 0)\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \]

what is its state at time \( t \)? If we perform measurements of the kinetic energy and potential energy of the system at time \( t \), what do we get? Do they depend on time? (10 Calories)

8. Give one physical example of such quantum harmonic oscillators. Hint: one of such examples is a nuclear material. (10 Calories)
4 Homework 4

Readings:


4.1
What are the energies of an electron, a proton, a neutron, and a photon with wavelengths $\lambda = 1 \text{ Å}$ and 1 fm, respectively? (20 Calories)

4.2
What are the kinetic energy, velocity, and wavelength of thermal ($T = 300 \text{ K}$, room temperature) neutrons? (15 Calories) Why are thermal neutrons useful for materials studies? Give at least three reasons. (3 Calories)

4.3
What is the radius of gyration $R_g$ of a uniformly distributed spherical nucleus with a radius $R$?
Note: $R_g = \langle r^2 \rangle^{\frac{1}{2}} = \left( \int \rho(r)r^2dV \right)^{\frac{1}{2}}$. (10 Calories)

4.4
What is the potential $V(r)$ of an electron inside and outside a uniformly distributed spherical nucleus with a radius $R$ and charge $Ze$? (15 Calories)

4.5
1. Compute the binding energy per nucleon $\frac{B}{A}$ for $^4\text{He}$, $^7\text{Li}$, $^{56}\text{Fe}$, and $^{238}\text{U}$. (8 Calories)
2. Compute the neutron separation energy $S_n$ for $^4\text{He}$, $^7\text{Li}$, $^{56}\text{Fe}$, and $^{238}\text{U}$. (8 Calories)
3. Compute the proton separation energy $S_p$ for $^4\text{He}$, $^7\text{Li}$, $^{56}\text{Fe}$, and $^{238}\text{U}$. (8 Calories)

4.6 *
Griffiths’s Quantum Mechanics: Page 123, Problem 3.22. (20 Calories)

4.7 *
Griffiths’s Quantum Mechanics: Page 124, Problem 3.23. (20 Calories)

4.8 *
Griffiths’s Quantum Mechanics: Page 128, Problem 3.37. (20 Calories)
4.9  *
Griffiths's Quantum Mechanics: Page 127, Problem 3.35. (60 Calories)
5 Homework 5

Readings:


5.1

1. In order to solve the 3-D time-independent Schrödinger equation for a 3-D isotropic potential \( V(r) \), we use separation of variables. Show that the ordinary differential equation of the radial function \( R(r) = \frac{u(r)}{r} \) is

\[
-\frac{\hbar^2}{2m} \frac{d^2 u(r)}{dr^2} + \left[ V(r) + \frac{\hbar^2}{2m} (l+1) \right] u(r) = Eu(r)
\]

(10 Calories)

2. Sketch the neutron-proton interaction \( V(r) \). (10 Calories)

3. Solve and sketch the bound \( s \)-wave \((l = 0)\) radial wave function for the deuteron. (20 Calories)

4. What is the binding energy of deuteron? How do you measure it experimentally? How does it compare with the average binding energy per nucleon \( B/A \)? (10 Calories)

5. What is the probability of finding the neutron or the proton outside the deuteron radius \( R \)? (10 Calories)

6. What is the radius of gyration \( R_g = \langle r^2 \rangle^{1/2} \) of the deuteron? (10 Calories)

5.2

Krane: Page 113, Problem 4.3 (40 Calories) The condition for the existence of a bound state in the square-well potential can be determined through the following steps:

(a) Using the complete normalized wave function, show that the expectation value of the potential energy is

\[
\langle V \rangle = \int \psi^* V \psi dv = -V_0 A^2 \left[ \frac{1}{2} R - \frac{1}{4k_1} \sin 2k_1 R \right]
\]

(10 Calories)

(b) Show that the expectation value of the kinetic energy is

\[
\langle T \rangle = \frac{\hbar^2}{2m} \int_0^\infty \left| \frac{\partial \psi}{\partial r} \right|^2 dv = \frac{\hbar^2}{2m} A^2 \left[ \frac{1}{2} k_1^2 R - \frac{1}{4} k_1 \sin 2k_1 R + \frac{1}{2} k_2 \sin^2 k_1 R \right]
\]

(10 Calories)

(c) Show that, for a bound state to exist, it must be true that \( \langle T \rangle < -\langle V \rangle \). (10 Calories)
(d) Finally, show that a bound state will exist only for $V_0 \geq \frac{\pi^2 \hbar^2}{8mR^2}$ and evaluate the minimum depth of the potential that gives a bound state of deuteron. (10 Calories)

(Note: This calculation is valid only in three-dimensional problems. In the one-dimensional square well (indeed, in all reasonably well-behaved attractive one-dimensional potentials) there is always at least one bound state. Only in the three-dimensional problems is there a critical depth for the existence of a bound state. See C. A. Kocher, *Am. J. Phys.* 45, 71 (1977).)

5.3 *

The Hamiltonian of a spin 1/2 system (e.g., a neutron, a proton, or an electron) is

$$\hat{H} = \epsilon_1 (|\uparrow\rangle \langle \uparrow| + |\downarrow\rangle \langle \downarrow|) + \epsilon_2 (|\uparrow\rangle \langle \downarrow| + |\downarrow\rangle \langle \uparrow|)$$

1. What is the matrix representation of $\hat{H}$ in the $|\uparrow\rangle$ and $|\downarrow\rangle$ basis? (10 Calories)

2. Find its eigenvalues and eigenvectors as linear combinations of $|\uparrow\rangle$ and $|\downarrow\rangle$. Note that $|\uparrow\rangle$ and $|\downarrow\rangle$ are not the energy eigenstates of this system. (10 Calories)

3. If the system starts out in state $|\uparrow\rangle$, what is its state at time $t$? (10 Calories)

5.4 *

What is the probability of finding the ground state electron of a hydrogen atom inside its nucleus? (10 Calories)
6 Homework 6

Readings:

6.1
Generally speaking, what are the common types of interaction of neutrons with matter? Explain each type. (10 Calories)

6.2
1. Explain the meaning of the scattering differential cross section \( \frac{d\sigma}{d\Omega} \). (10 Calories)
2. Using the neutron-proton interaction potential derived from the bound state of deuteron, what is the total neutron-proton scattering cross section we computed? What is the experimentally measured value of the total neutron-proton cross section? How do you explain the discrepancy? (10 Calories)
3. Why is water important for nuclear engineering? Give at least three reasons. (10 Calories)

6.3
Derive the Q-equation:

\[
Q = E_3 \left(1 + \frac{m_3}{m_4}\right) - E_1 \left(1 - \frac{m_1}{m_4}\right) - \frac{2}{m_4} \sqrt{m_1 m_3 E_1 E_3} \cos \theta
\]

(10 Calories)

6.4
Consider a two-particle, say neutron with mass \( m \) and a target nucleus with mass \( Am \), elastic scattering process. The energy of the incident neutron is \( E \). The target nucleus is at rest before the collision.

1. Derive the relation between the energy \( E' \) and the outgoing angle \( \theta \) of the scattered neutron in the Laboratory (L) coordination system. (10 Calories)
2. Show that in the Center-of-Mass (CM) coordination system the magnitude of the velocity each particle does not change before and after the collision. Only their directions change. (10 Calories)
3. Derive the relation between the energy \( E' \) and the outgoing angle \( \theta_c \) of the scattered neutron in the Center-of-Mass (L) coordination system. (10 Calories)
4. Derive the relation between $\theta$ and $\theta_c$:

$$\cos \theta = \frac{1 + A \cos \theta_c}{\sqrt{A^2 + 1 + 2A \cos \theta_c}}$$

(10 Calories)

6.5

1. Sketch the typical energy distribution of scattered neutrons $F(E \rightarrow E')$. Explain its physical significance. (10 Calories)

2. What’s the average energy loss of the neutrons. (10 Calories)

6.6

Sketch the energy-dependence of the total neutron scattering cross section $\sigma_s(E)$ for graphite and water. Explain the main features of the curves. (30 Calories)

6.7 *

Read Chapter 4 of Griffith’s *Quantum Mechanics* carefully and study how to solve the eigenstates of hydrogen atom. Summarize the solutions of the eigen wavefunctions, energy, and angular momentum. Explain their physical meanings. (No need to solve the differential equations.) (30 Calories)

6.8 *

1. Produce pseudo-color plots of the spherical harmonics $Y_l^m(\theta, \phi)$ on a sphere for $l \leq 3$ and all the allowed $m$. The color should be mapped to the values of $Y_l^m(\theta, \phi)$. (10 Calories)

2. Produce constant surface plots of the real, imaginary, and absolute values of the hydrogen atomic orbitals $\psi_{nlm}(r, \theta, \phi)$ for $n \leq 3$ and all the allowed $l$ and $m$. (10 Calories)

3. Produce pseudo-color plots of the probability density (2D cut along $z$ axis) of the hydrogen atomic orbitals $\psi_{nlm}(r, \theta, \phi)$ for $n \leq 3$ and all the allowed $l$ and $m$. The color should be mapped to the values of the probability density $|\psi_{nlm}(r, \theta, \phi)|^2$. (10 Calories)

Attach all of your codes.
7 Homework 7

Readings:

7.1 What are the thermal neutron transmission coefficients of 1 mm thick light and heavy water respectively? (20 Calories)

7.2
1. Explain Compton scattering. (10 Calories)
2. Derive the Compton shift formula:
   \[ \Delta \lambda = \lambda' - \lambda = \lambda_c (1 - \cos \theta) \]
   where \( \lambda_c = \frac{2 \pi \hbar}{m_e c} \) is the Compton wavelength. (20 Calories)
3. Show that the kinetic energy of the recoil electron (Compton electron) is
   \[ T = \hbar \omega \frac{\alpha (1 - \cos \theta)}{1 + \alpha (1 - \cos \theta)} \]
   where \( \alpha = \frac{\hbar \omega}{m_e c^2} \) is the ratio of the energy of the photon and the rest energy of the electron. (10 Calories)

7.3
1. Sketch the angular distribution of Compton scattering. Explain the main features of the curve. (15 Calories)
2. Sketch the energy distribution of Compton scattering. Explain the main features of the curve. (15 Calories)

7.4 What is the fate of an excited atom produced by the photoelectric effect? (15 Calories)

7.5 In the \( Z \) (the atomic number of the materials) vs \( E \) (the energy of the photon) diagram, sketch which regions are dominated by which processes. (15 Calories)
7.6
Sketch the energy dependence of the attenuation coefficient of photons. Explain the main features of the curve. (15 Calories)

7.7 *
Macroscopic Quantum World: Planck constant (reduced) $\hbar \approx 1 \times 10^{-34} \text{ J} \cdot \text{s}$ plays a fundamental role in quantum mechanics. Imagine that one day you are transported to another universe, where the reduced Planck constant is $10^{34}$ times larger, i.e. $\hbar \approx 1 \text{ J} \cdot \text{s}$. Let’s picture what strange phenomena you would expect.

1. First, derive the uncertainty principle
$$\sigma_A \sigma_B \geq \frac{1}{2} |\langle [A,B] \rangle|$$
and state its the significance. (10 Calories)

2. Suppose you have a jar of candies in this quantum world. When you open the jar, estimate the escape velocities of the candies. Do you need to be careful when opening the jar? (A typical weight of a candy is 1 gram. A typical size of a jar is 10 cm.) (10 Calories)

3. Use your imagination, describe another two strange phenomena you would expect in this quantum world. Be as quantitative as possible. (20 Calories)

7.8 *
1. Prove $[L_x, L_y] = i\hbar L_z$ (5 Calories)

2. Prove $[L^2, L_z] = 0$ (5 Calories)

3. Use the ladder operators, prove $L^2|l, m\rangle = \hbar^2 l(l + 1)|l, m\rangle$ and $L_z|l, m\rangle = \hbar m|l, m\rangle$ (20 Calories)

4. Prove $L_{\pm}|l, m\rangle = \hbar \sqrt{(l \mp m)(l \pm m + 1)}|l, m \pm 1\rangle$ (10 Calories)

7.9 *
Write down the angular momentum operator $\mathbf{L}$ in spherical coordinates. Show that
$$L^2 = -\hbar^2 \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right]$$
and
$$L_z = -i\hbar \frac{\partial}{\partial \phi}$$
Therefore, the eigen equations of them are indeed the angular equations obtained from the separation of variables. (20 Calories)
8 Homework 8

Readings:


8.1
Sketch the incident particle energy dependence of the stopping power. Explain the main features of the curve. (15 Calories)

8.2
Sketch the Bragg curve of an α particle in air. Explain the main features of the curve. (15 Calories)

8.3*
If the spin angular momentum of an electron ($\frac{1}{2} \hbar$ along the z-axis) comes from the rotation of a classical sphere of radius $r_c$, calculate the linear speed of the equator in m/s. The $r_c$ is the so-called classical electron radius, which is obtained by assuming the Coulomb energy is the same as the mass energy: $\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_c} = mc^2$ (20 Calories)

8.4*
Suppose a neutron (or an electron, or a proton) is in the spin state $A \begin{pmatrix} i \\ 1 \end{pmatrix}$.

1. What is the normalization factor $A$? (5 Calories)

2. If we measure $S_x$, what values do we get? What is the probability of each value? (5 Calories)

3. If we measure $S_y$, what values do we get? What is the probability of each value? (5 Calories)

4. If we measure $S_z$, what values do we get? What is the probability of each value? (5 Calories)

5. What are the expectation values of $\langle S_x \rangle$, $\langle S_x^2 \rangle$, $\langle S_y \rangle$, $\langle S_y^2 \rangle$, $\langle S_z \rangle$, and $\langle S_z^2 \rangle$? Verify the uncertainty relations. (20 Calories)

8.5*
Construct the spin matrices $S_x$, $S_y$, and $S_z$ for a particle with spin 1 (for example, a deuteron). (30 Calories)